Stable Oscillations of a Spatially Chaotic Wave Function in a Microstadium Laser

Takahisa Harayama,[1] Peter Davis,[1] and Kensuke S. Ikeda[1,2]

[1]*ATR Adaptive Communications Research Laboratories, 2-2-2 Hikaridai, Seika-cho, Soraku-gun, Kyoto 619-0228, Japan*
[2]*Department of Physics, Ritsumeikan University, 1916 Noji-cho, Kusatsu, Shiga 525, Japan*

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Laser action on a single spatially chaotic wave function is obtained as a final stable state in a nonlinear dynamical model of a stadium shaped resonant cavity with an active medium. The stable single-mode lasing state corresponds to a particular metastable resonance of the cavity which wins a competition among multiple modes with positive net linear gain and has a distinct lasing threshold.

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The nonlinear interaction between the light field and the lasing medium has been an open problem for the studies of lasing in two-dimensional resonant cavities such as microdisk lasers. Previously, only the correspondence between rays and linear wave modes has been used to understand the lasing in circularly symmetric cavities [1]. The whispering gallery modes observed to lase in microdisk lasers correspond to a set of ray trajectories which never escape the cavity. A ray description has also been applied to the lasing mechanism of the deformed cavity proposed for spoiling high confinement of light in the microdisk and getting some laser light out of the disk [2,3]. The ray-dynamical trajectories in these deformed cavities become chaotic, but not fully chaotic, and still have stable periodic orbits, and, hence, it is still possible to understand the relation between shape and optical confinement properties in terms of ray trajectories [2,3].

This method of understanding cavity modes in terms of ray trajectories becomes much less effective in fully chaotic cavities. Fully chaotic cavities do not have any stable ray trajectories, and the optical modes inside the cavity are typically complicated wave functions which do not have a simple description in terms of a periodic or quasiperiodic ray trajectory inside the cavity [4]. Accordingly, it is difficult to predict what type of modes will lase, and whether they can be expected to lase stably in fully chaotic cavities.

In this Letter, we show that a fully nonlinear dynamical treatment of lasing in a fully chaotic cavity gives single-mode oscillation of a spatially chaotic wave function. For the cavity shape, we chose Bunimovich's stadium as shown in Fig. 1, which is well known as a shape which has been exactly proven to be fully chaotic, and has been a popular model in research on classical and quantum chaos [5–7]. For the laser model, we use the Schrödinger-Bloch model [8]. The Schrödinger-Bloch model is an approximation of the Maxwell-Bloch model taking into consideration the nonlinear interaction between the light field and the lasing medium, and was originally introduced for the study of microdisk lasers. We have checked that this model reproduces the lasing characteristics of the conventional microdisk lasers [8]. Here we extend this model to a microstadium laser and derive the criterion for the stable oscillation of the resonance modes. We also show that the spatially chaotic wave function of the stationary lasing oscillation excellently corresponds to the quasistationary state of the resonance obtained by an extended boundary element method.

First, let us briefly review how to simulate the light field in a two-dimensional microcavity with an active medium [8,9]. We assume a two-dimensional optical waveguide whose thickness is less than the wavelength, and which contains a lasing medium. We also assume the waveguide is wide in the xy directions and thin in the z direction, and that the refractive index suddenly changes on the edge of the cavity.

In order to describe the active gain medium in interaction with the light field, we use the well-known two-level model, the optical-Bloch equation. The two-level medium in turn contributes to the Maxwell equation as

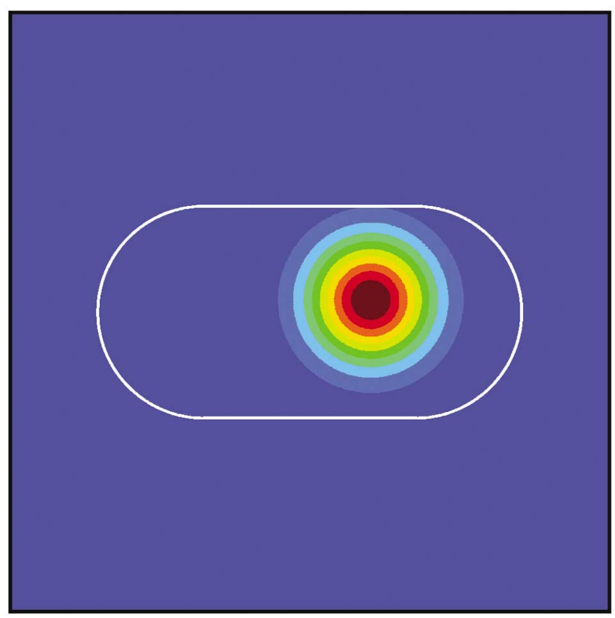

FIG. 1 (color). The initial state of the light field. The final stationary state does not depend on the initial state. The white curve denotes the stadium cavity.

the radiation source term. Thus, the time evolution of the TM wave of the light field obeys the two-dimensional Maxwell-Bloch equation [9].

Let ω_0 be the transition frequency of the two-level medium and suppose $\tilde{E}$ and $\tilde{\rho}$ are the slowly varying envelope of the electric field and the polarization field. Then, the slowly varying envelope approximation for time yields the following set of equations composed of a Schrödinger equation for $\tilde{E}$:

$$\frac{\partial \tilde{E}}{\partial t} = \frac{i}{2}\left(\nabla_{xy}^2 + \frac{n^2}{n_{\mathrm{in}}^2}\right)\tilde{E} - \alpha_L(x, y)\tilde{E} + \frac{2\pi N\kappa\hbar}{\epsilon}\tilde{\rho}, \quad (1)$$

and the optical-Bloch equations,

$$\frac{\partial \tilde{\rho}}{\partial t} = -\tilde{\gamma}_\perp \tilde{\rho} + \tilde{\kappa} W \tilde{E}, \quad (2)$$

$$\frac{\partial W}{\partial t} = -\tilde{\gamma}_\parallel (W - W_\infty) - 2\tilde{\kappa}(\tilde{E}\tilde{\rho}^* + \tilde{E}^*\tilde{\rho}), \quad (3)$$

where space and time are made dimensionless by the scale transformation $(n_{\mathrm{in}}\omega_0 x/c, n_{\mathrm{in}}\omega_0 y/c) \rightarrow (x, y)$ and $t\omega_0 \rightarrow t$, respectively. In the above, the refractive index n equals n_{in} inside the cavity and n_{out} outside the cavity, and $\alpha_L(x, y)$ is the linear absorption coefficient, which is the constant α_L inside the cavity and zero outside the cavity. In the Bloch equation, W is the population inversion component, and the two (dimensionless) relaxation parameters $\tilde{\gamma}_\perp$ and $\tilde{\gamma}_\parallel$ are the transversal relaxation rate and the longitudinal relaxation rate, respectively, and W_∞ is the external pumping parameter. $\tilde{\kappa}$ is the dimensionless coupling strength. We refer to this as the 2D Schrödinger-Bloch model.

Now we derive the criterion for stable oscillation solutions of the Schrödinger-Bloch equation. We assume that the light field and polarization oscillate as $\tilde{E}(\mathbf{r}, t) = e^{-i\Delta t}\hat{E}(\mathbf{r})$ and $\tilde{\rho}(\mathbf{r}, t) = e^{-i\Delta t}\hat{\rho}(\mathbf{r})$, where Δ is a real number and denotes the steady-state oscillation frequency while the population inversion does not oscillate and $W(\mathbf{r}, t) = W(\mathbf{r})$. Then we obtain a nonlinear steady-state equation,

$$\left(\nabla_{xy}^2 + \frac{n^2}{n_{\mathrm{in}}^2} + 2\Delta\right)\hat{E} = \frac{i2\alpha(\Delta)\tilde{\gamma}_\parallel W_\infty}{\tilde{\gamma}_\parallel + \frac{4\tilde{\kappa}^2\tilde{\gamma}_\perp}{\tilde{\gamma}_\perp^2 + \Delta^2}|\hat{E}|^2}\hat{E} - \alpha_L(x, y)\hat{E}. \quad (4)$$

Here,

$$\alpha(\Delta) = \frac{\alpha_0}{1 - i\Delta/\tilde{\gamma}_\perp}, \quad (5)$$

where

$$\alpha_0 = \frac{2\pi N\kappa\hbar}{\epsilon}\frac{\tilde{\kappa}}{\tilde{\gamma}_\perp}. \quad (6)$$

The real part of $\alpha(\Delta)$ is the linear laser gain, which is maximum for $\Delta = 0$.

The difference between Eq. (4) and the usual linear Schrödinger-Helmholtz equation is the terms, in the right-hand side of Eq. (4), which contain the linear loss due to absorption $\alpha_L(x, y)$ and the laser gain $\alpha(\Delta)$. Without these terms, we can obtain the quasistationary states, i.e., the resonances of the cavity as the eigenmodes of the Schrödinger-Helmholtz equation. The resonance frequencies are obtained as the complex-valued eigenvalues Δ_j. The real part $\mathrm{Re}\,\Delta_j$ of the eigenvalue represents the oscillation frequency of the quasistationary state. The imaginary part $\mathrm{Im}\,\Delta_j (<0)$ of the eigenvalue represents the decay rate of the quasistationary state.

If we include the first order correction of Δ_j due to the presence of the linear absorption loss and gain terms in Eq. (4), Δ_j is modified as

$$\Delta_j = \mathrm{Re}\,\Delta_j - i\alpha_{\mathrm{tot}} + i\alpha(\mathrm{Re}\,\Delta_j), \quad (7)$$

where $\alpha_{\mathrm{tot}} = -\mathrm{Im}\,\Delta_j + \alpha_L$ stands for the total linear loss, and the imaginary part of the last term is the linear laser gain.

When the gain term, $\mathrm{Re}\,\alpha(\mathrm{Re}\,\Delta_j)$, exceeds the total linear loss, the corresponding mode grows exponentially and can lase. Accordingly, we obtain the condition for an eigenmode to lase:

$$\frac{\alpha_0}{1 + \mathrm{Re}\,\Delta_j^2/\tilde{\gamma}_\perp^2} > -\mathrm{Im}\,\Delta_j + \alpha_L. \quad (8)$$

Such a linear description is correct only when the field intensity is weak. How a mode behaves asymptotically is answered only by the full nonlinear analysis of the Schrödinger-Bloch equations (1)–(3). The whispering gallery modes have been obtained as solutions of Eq. (4) in the case of a circular resonant cavity, and their stabilities have been checked by the dynamical simulation of Eqs. (1)–(3) [9,10]. However, in the case of the resonant cavity of a stadium shape, it is very difficult to solve Eq. (4). Therefore, the dynamical approach is extremely important in the case of a microstadium laser.

We simulated the time evolution of the light field starting from an initial condition of a Gaussian wave packet of width 5.0 centered at the point (5.0,1.0) (with respect to the center of the cavity) in a stadium shape cavity which consists of two half circles of the radius $R = 49/4\sqrt{2} \approx 8.75$ and two flat lines of the length $2R$ as shown in Fig. 1. We set the refractive index inside and outside the stadium $n_{\mathrm{in}} = 2$ and $n_{\mathrm{out}} = 1$, respectively. The other parameters are reported to be as follows: $\tilde{\gamma}_\parallel = 0.03$, $\tilde{\gamma}_\perp = 0.06$, $\epsilon = 4.0$, $\alpha_L = 0.004$, $N\tilde{\kappa}\hbar\omega_0 = N\kappa\hbar = 0.5$. In this Letter, all the quantities are made dimensionless. If we would assume the vacuum wavelength of the lasing mode is 0.86 μm, the length of the flat side of the stadium would be 1.2 μm.

We extended the boundary element method in order to solve the linear Schrödinger-Helmholtz equation under the boundary condition that the refractive index suddenly changes on the edge of the cavity. The resonances obtained by the extended boundary element method are shown in Fig. 2. The lasing condition (8) is evaluated in the case that the pumping power $W_\infty = 0.003$. The single and double circles in Fig. 2 satisfy the lasing condition (8)

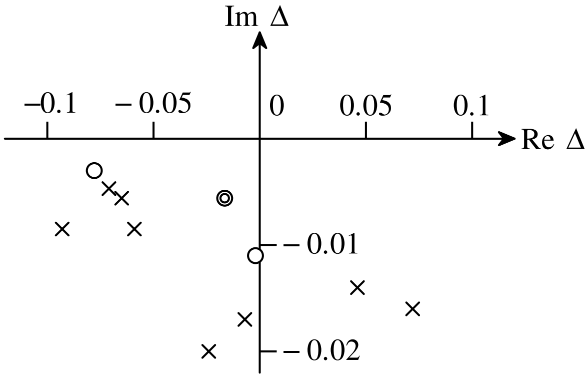

FIG. 2. Resonances of a microstadium cavity. The double circle denotes the resonance corresponding to the stable oscillation obtained as a result of time evolution simulations. The single circles have enough lasing gain to exceed the linear loss and satisfy the condition (8), but they lose mode competition with the resonance of the double circle in the course of time evolution. The crosses correspond to the resonances which do not satisfy the condition (8).

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In order to carry out the dynamical simulation, we used the symplectic integrator method for the Schrödinger equation and the Euler method for the Bloch equations [11]. The total light intensity inside the stadium is small at first; however, it grows exponentially and saturates to be a constant as shown in Fig. 3. In Fig. 4, we show the power spectrum of the time evolution of the light field in the time region after the saturation. It shows a sharp peak precisely corresponding to the real part of the resonance frequency of the double circle in Fig. 2. From the results of Figs. 3 and 4, one can see that the laser action occurs on the resonance marked by the double circle in Fig. 2.

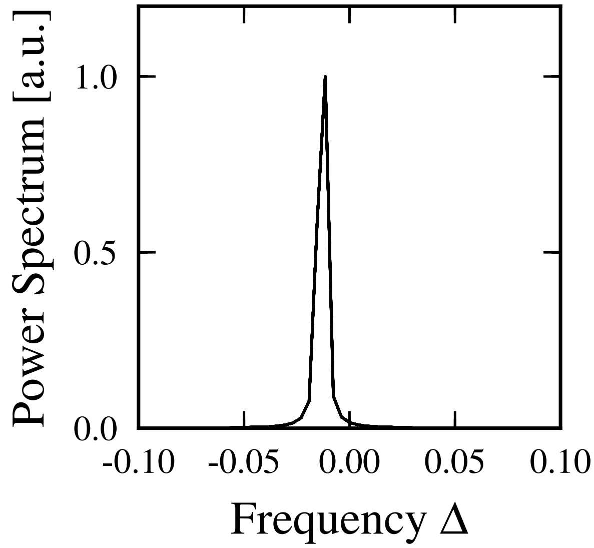

FIG. 4. The power spectrum obtained from the time evolution of the light field in the time region of a stable oscillation. The oscillation frequency corresponds well to that of the resonance denoted by the double circle in Fig. 2.

Indeed, the spatial wave function of the final stable state shown in Fig. 5 excellently corresponds to the wave function of the resonance marked by the double circle in Fig. 2 shown in Fig. 6 obtained by an extended boundary element method. Therefore, we conclude that only the resonance mode of the double circle in Fig. 2 wins the mode competition with the other resonance modes and can lase. We were unable to identify closed ray trajectories corresponding either to the lasing mode in Fig. 5 or to the other quasibound states in Fig. 2, and

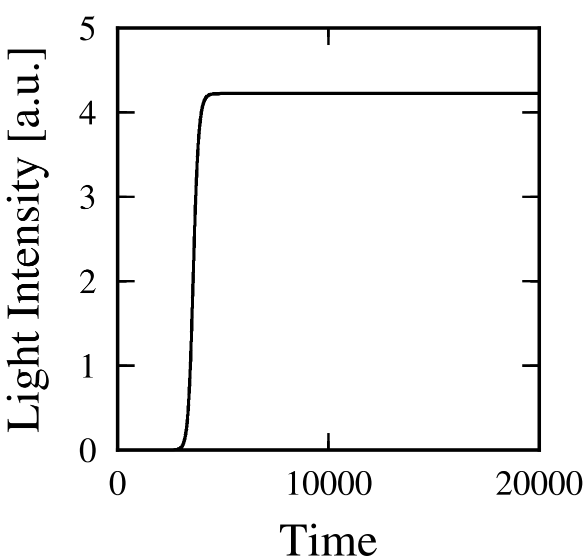

FIG. 3. Time evolution of the total light intensity inside the stadium cavity. The intensity grows exponentially and finally saturates to be constant, which means that the light field finally becomes a stationary oscillating state.

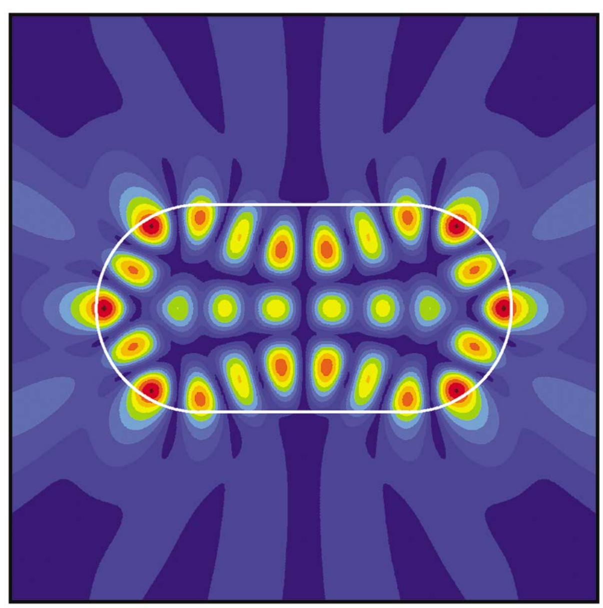

FIG. 5 (color). The final stable oscillation of a spatially chaotic wave. It oscillates with the peak frequency shown in Fig. 4.

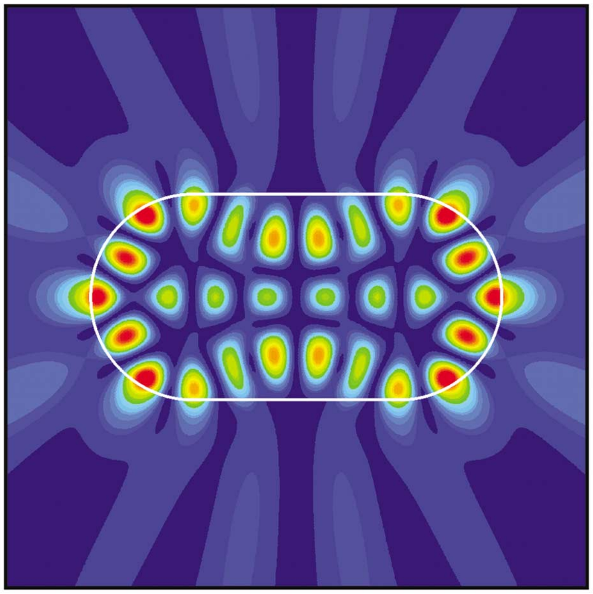

FIG. 6 (color). The wave function of the metastable resonance of the double circle in Fig. 2. This spatially chaotic wave function is a solution of the linear Schrödinger-Helmholtz equation and excellently corresponds to Fig. 5.

therefore we were not able to use total internal reflection as a criterion to predict which resonance wave function would lase. Our dynamical simulation shows that around the threshold single-mode lasing always occurs on the resonance mode which satisfies the inequality (8) best. However, in general, only the fully nonlinear dynamical simulation gives the lasing characteristics.

Figure 7 shows the total light intensity inside the stadium cavity as a function of the pumping power W_∞. When the pumping power W_∞ is smaller than 0.001 64, the resonance mode of the double circle in Fig. 2 does not satisfy the lasing condition (8) and it cannot lase. The threshold phenomenon is clearly seen, as in the case of conventional lasers. The actual lasing threshold is around 0.002, slightly larger than the value 0.001 64 predicted by the lasing condition (8).

Finally, let us remark that recently a semiconductor laser with the microstadium shape has been actually fabricated by using an MBE-grown GRIN-SCH-SQW GaAs/AlGaAs structure and a reactive-ion-etching technique [12]. The length of the flat side of the stadium is 30 μm. On the other hand, the lasing wavelength is about 0.26 μm inside the microstadium cavity, which is much smaller than the size of the cavity. Consequently, our nonlinear dynamical method described above cannot be applied to this semiconductor microstadium laser diode with the present computation power. However, the extended boundary element method can be used to calculate the resonances and identify which resonance mode corresponds to the observed wavelength and far field pattern. The relation between the injection current and the intensity of the output light from this microstadium laser shows a threshold phenomena characteristic of onset of

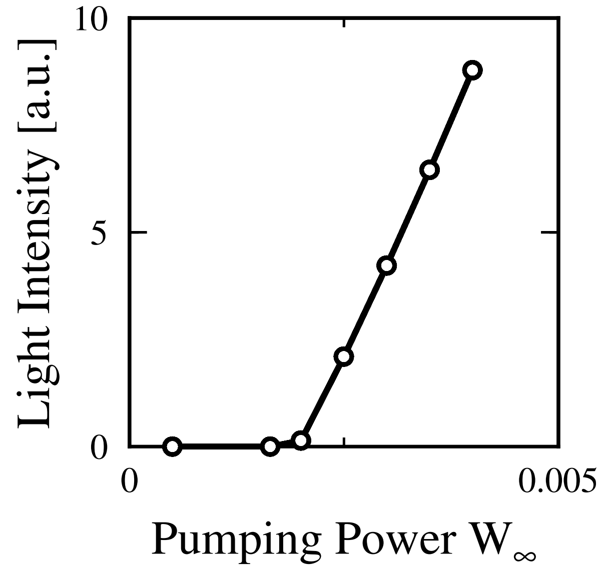

FIG. 7. Light intensity vs pumping power. A threshold phenomenon is observed as in the case of usual lasers.

lasing. A sharp narrowing of the optical spectrum above threshold is also observed. Therefore, the experimental observation of lasing in the real semiconductor microstadium has also demonstrated that lasing is possible in fully chaotic cavities.

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[1] *Optical Processes in Microcavities*, edited by R. K. Chang and A. J. Campillo (World Scientific, Singapore, 1996).
[2] J. U. Nöckel and A. D. Stone, Nature (London) **385**, 45 (1997).
[3] C. Gmachl, F. Capasso, E. E. Narimanov, J. U. Nöckel, A. D. Stone, J. Faist, D. L. Sivco, and A. Y. Cho, Science **280**, 1556 (1998).
[4] M. C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (Springer-Verlag, Berlin, 1990).
[5] L. A. Bunimovich, Commun. Math. Phys. **65**, 295 (1979).
[6] S.W. McDonald and A. N. Kaufman, Phys. Rev. A **37**, 3067 (1988).
[7] H.-J. Stöckmann, *Quantum Chaos: An Introduction* (Cambridge University Press, Cambridge, England, 1999).
[8] M. Hirayama, M. Kubota, T. Harayama, P. Davis, and K. S. Ikeda (unpublished).
[9] T. Harayama, P. Davis, and K. S. Ikeda, Prog. Theor. Phys. Suppl. **139**, 363 (2000).
[10] T. Harayama, P. Davis, and K. S. Ikeda, Phys. Rev. Lett. **82**, 3803 (1999).
[11] K. Takahashi and K. S. Ikeda, J. Chem. Phys. **106**, 4463 (1997).
[12] T. Harayama, T. Fukushima, P. Davis, P. Vaccaro, T. Miyasaka, T. Nishimura, and T. Aida (unpublished).